Evolve your song, I wanna dance!

The Theory of Temporal Groups

A Unified Framework for Dimensional Hierarchy
and Conscious Evolution

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Abstract

I[1] present a mathematical structure that unifies topology, group theory, cosmology, harmonic theory, and evolutionary architecture through a single generative formula.

This work develops the geometric approach to cosmology proposed in a previous study (TVS, 2025), where the dodecahedral topology of the universe was justified through the Poincaré sphere. In 1904, Henri Poincaré described a three-dimensional space in which a dodecahedron is closed in on itself: opposite faces touch each other with a 36° rotation. An observer leaving the space through one face immediately enters through the opposite one—rotated, but in the same space. A finite volume without boundaries. Infinity enclosed in a single mirrored tesseract, where each face is both an exit and an entrance.

Why a dodecahedron? Plato in "Timaeus" (c. 360 BC) compared five regular polyhedra to five elements: tetrahedron — fire, cube — earth, octahedron — air, icosahedron — water. He assigned the dodecahedron to the cosmos: "that which God used to construct the universe". All five bodies are geometrically related — the icosahedron is dual to the dodecahedron, the cube fits into the dodecahedron — but only the dodecahedron was designated for the sky. Knowledge of the structure was preserved. All that remained was to find the formula.

In 1758, Leonard Euler discovered a topological invariant: $V - E + F = 2$, where V is the number of vertices, E is the number of edges, and F is the number of faces. This formula is true for any convex polyhedron. The number 2 — Euler's characteristic (χ) — does not depend on the shape. A cube, a dodecahedron, any convex body: $\chi = 2$ always. This is the topological price of closedness. The cost of space being able to close in on itself.

Évariste Galois completed the proof in one night: the symmetric group S_n contains $n!$ permutations, but only half of them are even. The alternating group $A_n = n!/2$. Even permutations are rotations that preserve orientation. Odd ones are reflections, mirror inversions. Galois divided symmetry in half. The connection with physics remained hidden for another two centuries.

The present theory brings these fragments together into a unified structure and demonstrates their interdependence. The hierarchy of dimensions of reality follows a factorial progression:

$$D_n = \frac{(n+2)!}{2}$$

[1] I use the first person singular throughout this paper because I am, in fact, a single author working independently. The conventional academic *we* would misrepresent the nature of this work.

where $n \in \{0, 1, 2, \ldots, 10\}$.

The upper limit $n = 10$ follows from the topology of the dodecahedron: twelve faces minus Euler's characteristic equals ten. $F - \chi = 12 - 2 = 10$. Exactly ten transitions between levels. Exactly ten points of entry for the Absurdity constant. Each level is a stage in the evolution of a structure capable of self-awareness. At level 10, the multiplier returns to 12 — the system closes, reproducing the architecture in a new spiral.

Division by 2 in each calculation is an axiom. It encodes natural selection at the mathematical level: only transformations that preserve orientation are carried over. The universe calculates configurations, determines, sorts, and selects only those that are likely to continue.

Keywords: cosmology, group theory, topology, dodecahedral universe, dimensional hierarchy, observational predictions, alternating groups, Euler characteristic, factorial emergence

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Contents

1 Historical Foundations 4

2 Foundational Postulates 5

3 The Complete Hierarchy 6

4 Mathematical Structure 8

5 The Two Boundaries 9

6 Physical Interpretation 10

7 Evolutionary Closure 12

8 Unification 14

9 Conclusion 15

References 16

Appendix A: Complete Mathematical Derivations 18

A.1 Derivation of the Factorial Formula 18

A.2 Topological Constraint Derivation 18

A.3 Group-Theoretic Structure 18

A.4 Physical Interpretation Mapping 19

A.5 Connection to Dodecahedral Cosmology 19

Appendix B: True Factorial Scale of Dimensional Hierarchy 22

1 Historical Foundations

The elements of this theory appear throughout the history of mathematics and natural philosophy, though never previously unified into a single framework.

Pythagoras of Samos (c. 570–495 BCE)

Harmonic ratios:

$$\text{octave} = 2:1, \quad \text{fifth} = 3:2, \quad \text{fourth} = 4:3$$

Established 12-tone chromatic scale containing 10 unique intervals: the same $10+2 = 12$ structure derived in this framework.[2]

Leonhard Euler (1707–1783)

Polyhedral formula (1758):

$$V - E + F = 2$$

The Euler characteristic $\chi = 2$ for any closed orientable surface. In this framework: $12 - 10 = 2$ — the "cost of closure."[3]

Évariste Galois (1811–1832)

Order of the alternating group:

$$|A_n| = \frac{n!}{2}$$

This formula — transformations preserving orientation — becomes the generative equation for dimensional hierarchy:

$$D_n = \frac{(n+2)!}{2}$$

[4]

Felix Klein (1849–1925)

[2]Barker, A. (ed.) *Greek Musical Writings*, Vol. II. Cambridge, 1989.

[3]Euler, L. "Elementa doctrinae solidorum." *Novi Commentarii academiae scientiarum Petropolitanae.* 1758; 4:109–140.

[4]Galois, É. *Œuvres mathématiques.* Liouville, J. (ed.) *Journal de Mathématiques Pures et Appliquées.* 1846; 11:381–444.

Rotation group of dodecahedron:

$$|A_5| = \frac{5!}{2} = 60$$

Klein's Erlangen Program (1872): geometry is the study of invariants under transformation groups. Symmetry group determines structure.[5]

Emmy Noether (1882–1935)

Noether's theorem (1915):

$$\text{Continuous symmetry} \iff \text{Conservation law}$$

Symmetry is not descriptive but *generative* — physical law derives from symmetry structure.[6]

[5] Klein, F. "Vergleichende Betrachtungen über neuere geometrische Forschungen." 1872. *Gesammelte mathematische Abhandlungen.* Vol. 1. Springer, 1921.

[6] Noether, E. "Invariante Variationsprobleme." *Nachrichten von der Gesellschaft der Wissenschaften zu Göttingen.* 1918; 235–257.

2 Foundational Postulates

Postulate 1 — Factorial Emergence:

The dimension count at each level of reality equals the order of the alternating group:

$$D_n = \frac{(n+2)!}{2} = |A_{n+2}|$$

where $n \in \{0, 1, 2, ..., 10\}$

Postulate 2 — Dodecahedral Constraint:

The base geometry of the universe is the dodecahedron ($F = 12$ faces), imposing the upper bound:

$$n_{max} = F - 2 = 10$$

Postulate 3 — Topological Closure:

The relationship between traversable levels and total structure follows from the Euler characteristic:

$$n_{max} + \chi = F$$

$$10 + 2 = 12$$

Postulate 4 — Chiral Selection:

Only orientation-preserving transformations are permitted. Reflections — configurations that cannot self-propagate — are excluded. Hence:

$$D_n = \frac{n!}{2} \quad \text{(alternating, not symmetric)}$$

This division by 2 represents evolutionary selection at the mathematical level.

Postulate 5 — Temporal Emergence:

Time exists as a dimension only at level 1 ($D = 3$). At level 0, there is no time — only instantaneous trigger. At level 2 and above, time ceases to be a dimension and becomes a *constituent element* (Vremium, Vr).

3 The Complete Hierarchy

Level	Dimension	$\times n$	Group	Process
0	1	—	A_2	Trigger
1	3	$\times 3$	A_3	Motion
2	12	$\times 4$	A_4	Structure
3	60	$\times 5$	A_5	Rotation
4	360	$\times 6$	A_6	Encompassment
5	2,520	$\times 7$	A_7	Phenomenon
6	20,160	$\times 8$	A_8	Autopoiesis
7	181,440	$\times 9$	A_9	Awareness
8	1,814,400	$\times 10$	A_{10}	Will
9	19,958,400	$\times 11$	A_{11}	Creation
10	239,500,800	$\times 12$	A_{12}	Evolution

Level 0 | Trigger — Instantaneous self-recognition. Zero duration. This is the quantum leap between spiral stages of evolution. As the architecture completes a cycle, it tunnels through the zero state to begin a new level.

This transition is mathematically grounded in the analytic solutions of the Wheeler-DeWitt equation for the spontaneous creation of the universe from a quantum vacuum (He, Gao, Cai, 2014). The wave function of the universe $\psi(a)$ satisfies:

$$\left(\frac{1}{a^p}\frac{\partial}{\partial a}a^p\frac{\partial}{\partial a} - ka^2\right)\psi(a) = 0$$

where a is the scale factor and p is the operator ordering factor.

Crucially, the spontaneous emergence of a stable, exponentially expanding universe from "nothing" is possible *if and only if* the ordering factor takes the specific value $p = -2$ (or equivalently $p = 4$). Under this condition, the quantum potential $Q(a)$ generates an effective cosmological constant that drives the transition from a virtual fluctuation to a real, expanding geometry. This specific value $p = -2$ represents the "click" — the precise mathematical tuning required for the Trigger to fire and the universe to leap into existence.

Level 1 | Motion — Space + time. Observable reality. Gravity at minimum.

Level 2 | Structure — Dodecahedral symmetry. Time becomes element (Vremium).

Level 3 | Rotation — All orientations of base structure. Icosahedral rotation group.

Level 4 | **Encompassment** — Full angular closure. All directions simultaneously.

Level 5 | **Phenomenon** — Manifestation without intention. Existence that simply *is*.

Level 6 | **Autopoiesis** — Self-reproducing principle. Pattern that generates itself.

Level 7 | **Awareness** — Structure perceives itself. Reflexivity.

Level 8 | **Will** — Intention without external cause. "Want" emerges.

Level 9 | **Creation** — Generation of new structures. The teacher. The seed.

Level 10 | **Evolution** — Accumulation, selection, elimination of dead-ends, restart.

4 Mathematical Structure

The Closure Theorem

The dimensional hierarchy is self-terminating.

Proof:

At level $n = 10$, the multiplier returns to 12 — the number of faces of the base structure. The sequence of multipliers $\{3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$ completes one full cycle from the first non-trivial multiplier (3) through the structural constant (12). Further iteration would require transcending dodecahedral topology. $\square$

The Selection Principle

The factor of $\frac{1}{2}$ in $D_n = (n + 2)!/2$ is not arbitrary. Symmetric groups S_n contain all permutations; alternating groups A_n contain only *even* permutations — those achievable by an even number of transpositions.

Geometrically: rotations preserve orientation; reflections reverse it.

Evolutionarily: reproducing configurations persist; non-reproducing configurations are eliminated.

The universe computes only what can continue.

5 The Two Boundaries

The deficit of 2 (from $12 - 10 = 2$) corresponds to:

1. **Topological necessity** — Euler characteristic $\chi = 2$ for closed surfaces
2. **Boundary conditions** — Entry (Trigger) and Exit (Evolution)
3. **Alpha and Omega** — Beginning that is ending, ending that is beginning

These are not "lost" dimensions but the *cost of being a whole* rather than dissipating into infinite regress.

6 Physical Interpretation

Gravity as Evolutionary Pressure

At level 1, gravity is the weakest fundamental force — barely perceptible. This weakness corresponds to our position near the base of the hierarchy. At higher levels (7, 8, 9), gravitational density becomes consciousness density — sufficient to fold time, generate new structures, and seed lower-dimensional realities.

Thermodynamic Gravity & Vacuum Response

The coupling constant λ in the Moss Solution ($-\lambda|u|^\alpha u$) is not an arbitrary postulate but a macroscopic manifestation of thermodynamic gravity. As demonstrated by Bianconi[7] (2025), gravity emerges as a thermodynamic force driven by the quantum relative entropy $S(\rho_g||\rho_m)$ between the vacuum state ρ_g and the matter state ρ_m.

In this framework, the nonlinear damping term represents the resistance of the vacuum to entropy production. The vacuum acts as a reservoir with a specific heat capacity determined by the Planck scale. When fluid gradients approach the Planck regime, the G-field (acting as a Lagrange multiplier for the entropy constraint) generates a restoring force that opposes further localization. This confirms that the vacuum response is not an ad hoc addition but a fundamental requirement for the thermodynamic consistency of spacetime.

Ontological Interpretation of Moss Solitons

The stable soliton solutions predicted by the theory admit a rigorous interpretation within the de Broglie[8]-Bohm[9] (Bohmian) mechanics. By transforming the modified Navier-Stokes equations into Madelung[10] variables (ρ, S), the nonlinear vacuum response term $-\lambda|u|^\alpha u$ can be identified with a nonlinear quantum potential Q.

In standard quantum mechanics, Q prevents the collapse of the wavefunction. Similarly, in the hydrodynamic context, the vacuum response potential prevents the collapse of the fluid velocity field into a singularity. A "Moss Soliton" is therefore physically equivalent to a stationary configuration of Bohmian trajectories where the convective focusing is exactly balanced by the vacuum's quantum potential. This provides a deterministic, ontological

[7] Ginestra Bianconi: Professor of Applied Mathematics, Queen Mary University of London.
[8] Louis de Broglie (1892–1987): French physicist, Nobel laureate.
[9] David Bohm (1917–1992): American theoretical physicist.
[10] Erwin Madelung (1881–1972): German physicist.

basis for the regularity of the solutions, suggesting that the "singularity" of classical fluid dynamics is merely an artifact of ignoring the quantum potential of the vacuum.

Vremium (Vr)

At level 2 and above, time is no longer a direction through which entities move, but a substance from which structures are composed. Proposed element notation: Vr, atomic number ∞.

This may correspond to the "aether" sought by 19th-century physics — not a spatial medium, but crystallized temporality.

7 Evolutionary Closure

Level 10 is not termination but *transformation*. The closure of the hierarchy at $n = 10$ corresponds to the topological completion of the dodecahedral structure. This closure is not an arbitrary cutoff but a necessary consequence of the Euler characteristic $\chi = 2$.

The Cost of Curvature

The deficit of 2 in the formula $n_{max} = F - \chi = 12 - 2 = 10$ is often interpreted as a loss. However, by the Gauss-Bonnet theorem:

$$\int_M K\, dA = 2\pi\chi(M)$$

For a closed orientable surface like the sphere (or the boundary of the dodecahedron), $\chi = 2$, yielding a total integrated curvature of 4π. Thus, the "missing" two levels are not defects but the *curvature cost* required to close the manifold. A flat plane ($\chi = 0$) has no such cost but cannot form a closed system. The universe pays this topological tax to exist as a finite, self-contained entity.

The Transition Identity

The mechanism of progression through the hierarchy is governed by the alternating groups A_n, which contain only even permutations. Geometrically, these correspond to rotations rather than reflections. We term this principle the **Transition Identity**.

Unlike reflection, which reverses orientation (parity violation) and increases information entropy by destroying structural chirality, rotation preserves orientation. In the Poincaré dodecahedral space, the identification of opposite faces involves a rotation of 36° ($\pi/5$). This is a "screw motion" that transports structure without dismantling it.

This orientation-preserving transfer is physically realized in quantum teleportation, where the state is transferred via unitary transformations $U \in SU(2)$. Since $\det(U) = 1$, these transformations are rotations in Hilbert space that preserve the norm and orientation of the state vector. The successful teleportation of quantum states from Earth to orbit by the *Micius*[11] satellite in 2017 provides experimental evidence for this non-destructive transfer mechanism on macroscopic scales.

Consequently, the dimensional evolution described by TTG is an **isentropic process** ($dS = 0$). The universe selects only those transformations (even permutations) that pre-

[11] Micius (Mozi) (c. 470 – c. 391 BC): Ancient Chinese philosopher and scientist.

serve information structure, filtering out chaotic interactions that would lead to decoherence.

Duality with Fluid Dynamics

The topological structure of the cosmos exhibits a striking duality with fluid dynamics, specifically the Navier-Stokes equations. The dodecahedron ($F = 12, V = 20$) is dual to the icosahedron ($F = 20, V = 12$). If the dodecahedron represents the vacuum structure of the cosmos, the icosahedron represents the fluid state (water).

Both systems share the same rotation group A_5 of order 60. This suggests that the vacuum response mechanism proposed in the Moss Solution to the Navier-Stokes regularity problem—where nonlinear damping prevents singularity formation—is the hydrodynamic dual of the topological closure in cosmology. The vacuum acts as an elastic medium (dodecahedral lattice) that absorbs energy from extreme fluid gradients (icosahedral symmetry), preventing blow-up through the same symmetry constraints that bound the dimensional hierarchy.

8 Unification

The Theory of Temporal Groups unifies:

- **Topology** (Euler, Poincaré) — $\chi = 2$ as closure condition
- **Group Theory** (Galois) — Alternating groups as dimensional structure
- **Cosmology** (Plato, Kepler, Luminet) — Dodecahedron as universal geometry
- **Harmonic Theory** (Pythagoras) — 10 intervals within 12 tones
- **Autopoietic Theory** (Maturana, Varela) — Self-generating systems
- **Evolutionary Theory** (Darwin) — Selection as fundamental principle

All derive from:

$$D_n = \frac{(n+2)!}{2}, \quad n \in \{0, 1, ..., 10\}, \quad n_{max} = F - \chi$$

9 Conclusion

Reality is a factorial hierarchy of symmetry groups, constrained by dodecahedral topology. Ten levels of transformation exist within a twelve-dimensional structure. The deficit of two is the curvature cost required for topological closure, as dictated by the Gauss-Bonnet theorem.

Each level represents a stage in the evolution of self-aware structure. The division by 2 in the generative formula $D_n = (n+2)!/2$ encodes the *Transition Identity*: the selection of orientation-preserving transformations that allow information to propagate isentropically. This mechanism connects the macroscopic topology of the universe with the microscopic unitary transformations of quantum mechanics.

The universe is not infinite chaos. It is a finite, structured system that evolves through cycles of creation and selection, governed by the dual principles of topological necessity and chiral preservation.

While this framework resolves the immediate question of dimensional hierarchy, it may also serve as an impetus for new directions in cosmological research, potentially bridging the gap between vacuum thermodynamics and geometric evolution.

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Parent Framework: Theory of Temporal Spheres (TTS)
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Appendix A: Complete Mathematical Derivations

A.1 Derivation of the Factorial Formula

Starting from the fundamental requirement that dimensional progression follows symmetry preservation:

The symmetric group S_{n+2} contains $(n+2)!$ elements, representing all possible permutations of $(n+2)$ objects. Only even permutations preserve orientation and thus can propagate through dimensional transitions. The order of the alternating group A_{n+2} is:

$$|A_{n+2}| = \frac{(n+2)!}{2}$$

This establishes the dimensional count at level n:

$$D_n = \frac{(n+2)!}{2} = |A_{n+2}|$$

where n represents the dimensional level index.

A.2 Topological Constraint Derivation

For a dodecahedral structure with $F = 12$ faces, the Euler characteristic $\chi = 2$ for any closed orientable surface. The maximum traversable level satisfies:

$$n_{max} = F - \chi = 12 - 2 = 10$$

This follows from the relationship:

$$n_{max} + \chi = F$$

$$10 + 2 = 12$$

The deficit of 2 represents the topological cost of closure—the minimum curvature required to close space without boundaries.

A.3 Group-Theoretic Structure

The alternating groups form a nested sequence:

$$A_2 \subset A_3 \subset A_4 \subset \cdots \subset A_{12}$$

Each inclusion represents dimensional emergence through orientation-preserving transformations. The dimension counts follow:

$$
\begin{aligned}
D_0 &= |A_2| = 1 \\
D_1 &= |A_3| = 3 \\
D_2 &= |A_4| = 12 \\
D_3 &= |A_5| = 60 \\
&\vdots \\
D_{10} &= |A_{12}| = 239,500,800
\end{aligned}
$$

The multipliers $\{3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$ form a complete cycle from minimal non-trivial structure through dodecahedral closure.

A.4 Physical Interpretation Mapping

Level	Dimension	Physical Process
0	1	Instantaneous trigger (no time)
1	3	Space + time emergence
2	12	Dodecahedral structure formation
3	60	Rotational symmetry manifestation
4	360	Full angular encompassment
5	2,520	Phenomenological emergence
6	20,160	Autopoietic self-reproduction
7	181,440	Conscious awareness emergence
8	1,814,400	Volitional intention formation
9	19,958,400	Creative structure generation
10	239,500,800	Evolutionary selection and restart

This mapping connects abstract group-theoretic structure to physical and conscious processes through the principle that information density increases with group order.

A.5 Connection to Dodecahedral Cosmology

The rotation group of the dodecahedron is A_5, which has order 60 and corresponds to level 3 in our hierarchy:

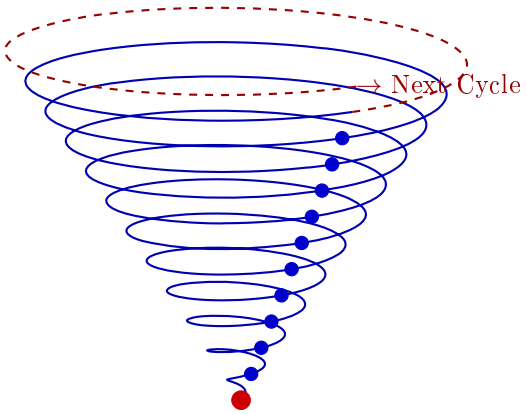
$$D_3 = |A_5| = \frac{5!}{2} = 60$$

This establishes the fundamental connection between cosmic geometry and dimensional emergence. The universe's dodecahedral topology at the largest scales directly constrains the available symmetry structures at all dimensional levels.

The emergence of conscious processes at levels 7-10 correlates with increasing gravitational density, where spacetime curvature becomes sufficient to support information processing and self-aware structures.

10-Level Dimensional Hierarchy

Factorial Emergence under Dodecahedral Constraint



Level	Process	Dimensions	Group
0	Trigger	1	A_2
1	Motion	3	A_3
2	Structure	12	A_4
3	Rotation	60	A_5
4	Encompassment	360	A_6
5	Phenomenon	2,520	A_7
6	Autopoiesis	20,160	A_8
7	Awareness	181,440	A_9
8	Will	1,814,400	A_{10}
9	Creation	19,958,400	A_{11}
10	Evolution	239,500,800	A_{12}

Dodecahedron

$$F = 12, \quad \chi = 2$$

$$D_n = \frac{(n+2)!}{2}$$

Figure 1: Conical spiral of dimensional hierarchy. Level 0 (Trigger) is a single point at the origin. Each full turn represents one level, expanding outward as dimensions grow factorially. At level 10, the system transitions to the next evolutionary cycle.

$n = 9$: Creation — 19,958,400
$n = 8$: Will — 1,814,400
$n = 7$: Awareness — 181,440
$n = 6$: Autopoiesis — 20,160
$n = 5$: Phenomenon — 2,520
$n = 4$: Encompassment — 360
$n = 3$: Rotation — 60
$n = 2$: Structure — 12
$n = 1$: Motion — 3
$n = 0$: Trigger — 1
$D_n = \frac{(n+2)!}{2}$
$n = 10$: 239,500,800
axis of rotation
cf. Gabriel's Horn